Relational and Algebraic Methods in Computer Science

Formalisms that admit calculational reasoning with a relation-algebraic flavour are finding applications in numerous areas of computer science. As a kind of "linear algebra of logic", these formalisms, including different kinds of allegories and Kleene algebras, enable methods of concise formalisation and reasoning in a "point-free" style with a high-level of abstraction. This facilitates not only high-confidence manual reasoning with good readability, but also automated tool support. These methods are the focus of a series of international conferences that started out using the name "Relational Methods in Computer Science" (RelMiCS), soon joined by "Applications of Kleene Algebra" (AKA); since 2011, the unified name "Relational and Algebraic Methods in Computer Science" (RAMiCS) has been adopted.

The 13th International Conference on Relational and Algebraic Methods in Computer Science (RAMiCS 2012) was held in Cambridge, UK, September 17–20, 2012; the proceedings of this conference have been published as Lecture Notes in Computer Science (LNCS), volume 7560, which contains 23 contributed papers, each reviewed by at least three referees. The programme committee of RAMiCS 2012 selected ten papers presented at the conference; we invited their authors and the three keynote speakers to submit an extended version for this special issue of the Journal of Logic and Algebraic Methods in Programming. Each of the ten substantially extended and revised submissions we received was again evaluated by at least two referees. The resulting collection of papers nicely illustrates the wide spectrum of different formalisms sharing that common "relation-algebraic flavour", and includes a number of different application areas.

Concurrent Kleene Algebra (CKA) has recently been introduced as a formalism for reasoning with a relation-algebraic flavour about concurrent processes. This application domain is currently dominated by the very different formalisms of concurrent separation logic — in their contribution *On the relation between Concurrent Separation Logic and Concurrent Kleene Algebra*, Peter W. O'Hearn, Rasmus L. Petersen, Jules Villard and Akbar Hussain explore commonalities and differences by comparing the proof theory of a concurrent separation logic with the algebraic laws of CKA.

Demonstrating a different way how separation logic may profit from interaction with Kleene algebra, Han-Hing Dang and Bernhard Möller extend separation logic in their paper *Extended Transitive Separation Logic* by introducing stronger separation predicates that take reachability into account, and add special features for linked data structure, altogether arriving at a program correctness calculus the power and elegance of which they demonstrate in correctness proofs of in-situ list reversal and a tree rotation operation, before demonstrating that their approach also deals elegantly with overlaid data structures.

In his contribution *Infinite executions of lazy and strict computations*, Walter Guttmann presents a unified semantics of iteration for lazy and strict computation models, with ability to reason about infinite and aborting executions, and expands the scope of established refinement theory. The Isabelle proofs for all algebraic results are publicly available.

A practical application of refinement is presented by Rudolf Berghammer and Sebastian Fischer in their paper *Combining Relation Algebra and Data Refinement to Develop Rectangle-Based Functional Programs for Reflexive-Transitive Closures*, where they achieve optimal asymptotic running-time complexity even with a simple successor-list implementation of relations in the purely functional programming language Haskell.

Point axioms are an essential tool for bridging the gap between quantifier-free relation-algebraic reasoning, and element-level predicate-logic reasoning; in their contribution *Point Axioms and Related Conditions in Dedekind Categories*, Hitoshi Furusawa and Yasuo Kawahara explicate the logical relationships among various such axioms in the setting of Dedekind Categories, which generalises abstract relation algebras in a way that also encompasses fuzzy relations.

Kleene algebras can be seen as a further generalisation of relation algebra, with the useful property that their equational theory is decidable. In the paper *Deciding Kleene Algebra Terms Equivalence in Coq*, Nelma Moreira, David Pereira, and Simão Melo de Sousa present a mechanically verified and publicly available decision procedure for equivalence of expressions in Kleene algebra, implemented using partial derivatives. They include a generalisation to check also expressions equivalence in Kleene algebra with tests (KAT), and make these decision procedures available in Coq tactics. KAT encompasses propositional Hoare

logic, and first applications of these tactics to automated program verification are reported.

Kleene algebras are closely related also to ω-regular algebras, which in addition capture infinite iteration, and for which Georg Struth, Michael R. Laurence, and James Cranch present improved axiomatisations, and prove new *Completeness Results for Omega-Regular Algebras*.

By removing iteration operators from ω-regular algebras, one essentially end up with complete idempotent left semirings; in their paper *Multirelational Representation Theorems for Complete Idempotent Left Semirings*, Hitoshi Furusawa and Koki Nishizawa use a concept of invertibility to distinguish three progressively "natural" levels of representability via multirelations.

In his contribution *Symmetric Heyting Relation Algebras with Applications to Hypergraphs*, John G. Stell proposes a variant of relation algebra with, instead of Boolean complement, only pseudocomplement and its dual, and with two separate left and right converse operations, motivated by investigations into mathematical morphology on hypergraphs.

For the application to search that takes user preferences into account, Bernhard Möller and Patrick Roocks prove algebraic laws about preference relations that can be used in query optimisation, in their contribution *An Algebra of Database Preferences*.

We are grateful to all the authors for submitting their papers and to the referees for their careful scrutiny. We are also most grateful to Jan Bergstra and Rocco De Nicola for making such a special issue once again possible. After 5 special issues of JLAP devoted to relational and algebraic methods in computer science appeared in 2006, 2008, 2010, 2011 and 2012, this is the first special issue of JLAMP on this topic. Special thanks go to Inge Bethke, Hilda Xu, Allwyn Richard Louis, Neethu Joseph, and Alberto Lluch Lafuente, for helping us with the technicalities and procedures of preparing this special issue. We thank the programme committee of RAMiCS 2012 for preselecting the papers, and the steering committee of the RAMiCS (formerly RelMiCS/AKA) conference series for its support. Last, but not least, we would like to thank the British Logic Colloquium and Winton Capital Management for sponsoring the RAMiCS 2012 conference.

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