Stronger Validity Criteria for Encoding Synchrony*

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Abstract. We analyse two translations from the synchronous into the asynchronous π-calculus, both without choice, that are often quoted as standard examples of valid encodings, showing that the asynchronous π-calculus is just as expressive as the synchronous one. We examine which of the quality criteria for encodings from the literature support the validity of these translations. Moreover, we prove their validity according to much stronger criteria than considered previously in the literature.

Keywords: Process calculi · expressiveness · translations · quality criteria for encodings · valid encodings · compositionality · operational correspondence · semantic equivalences · asynchronous π-calculus.

This paper is dedicated to Catuscia Palamidessi, on the occasion of her birthday. It has always been a big pleasure and inspiration to discuss with her.

1 Introduction

In the literature, many definitions are proposed of what it means for one system description language to encode another one. Each concept C of a valid encoding yields an ordering of system description languages with respect to expressive power: language $\mathscr{L}'$ is *at least as expressive as* language $\mathscr{L}$ (according to C), notation $\mathscr{L} \preceq_C \mathscr{L}'$, iff a valid encoding from $\mathscr{L}$ to $\mathscr{L}'$ exists. The concepts of a valid encoding themselves, the *validity criteria*, also can be ordered: criterion C is *stronger* than criterion D iff for each two system description languages $\mathscr{L}$ and $\mathscr{L}'$ one has

$$\mathscr{L} \preceq_C \mathscr{L}' \quad \Rightarrow \quad \mathscr{L} \preceq_D \mathscr{L}' .$$

Naturally, employing a stronger validity criterion constitutes a stronger claim that the target language is at least as expressive as the source language.

In this paper, we analyse two well-known translations from the synchronous into the asynchronous π-calculus, one by Boudol and one by Honda & Tokoro. Both are often quoted as standard examples of valid encodings. We examine which of the validity criteria from the literature support the validity of these encodings. Moreover, we prove the validity of these encodings according to much stronger criteria than considered previously in the literature.

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A *translation* $\mathscr{T}$ from (or *encoding* of) a language $\mathscr{L}$ into a language $\mathscr{L}'$ is a function from the $\mathscr{L}$-expressions to the $\mathscr{L}'$-expressions. The first formal definition of a *valid* encoding of one system description language into another stems from Boudol [4]. It is parametrised by the choice of a semantic equivalence $\sim$ that is meaningful for the source as well as the target language of the translation—and is required to be a congruence for both. Boudol in particular considers languages whose semantics are given in terms of labelled transition systems. Any semantic equivalence defined on labelled transition systems, such as strong bisimilarity, induces an equivalence on the expressions of such languages, and thus allows comparison of expressions from different languages of this kind. Boudol formulates two requirements for valid translations: (1) they should be *compositional*, and (2) for each source language expression P, its translation $\mathscr{T}(P)$—an expression in the target language—is semantically equivalent to P.

Successive generalisations of the definition of a valid encoding from Boudol [4] appear in [16,17,20]. These generalisations chiefly deal with languages that feature process variables, and that are interpreted in a semantic domain (such as labelled transition systems) where not every semantic value need be denotable by a closed term. The present paper, following [4] and most of the expressiveness literature, deals solely with *closed-term* languages, in which the distinction between syntax and semantic is effectively dropped by taking the domain of semantic values, in which the language is interpreted, to consist of the closed terms of the language. In this setting the only generalisation of the notion of a valid encoding from [16,17,20] over [4] is that Boudol's congruence requirement on the semantic equivalence up to which languages are compared is dropped. In [20] it is also shown that the requirement of compositionality can be dropped, as in the presence of process variables it is effectively implied by the requirement that semantic equivalence is preserved upon translation. But when dealing with languages without process variables, as in the present paper, it remains necessary to require compositionality separately.

A variant of the validity criterion from Boudol is the notion of *full abstraction*, employed in [38,40,41,30,29,2,11]. In this setting, instead of a single semantic equivalence $\sim$ that is meaningful for the source as well as the target language of the translation, two semantic equivalences $\sim_{\mathrm{S}}$ and $\sim_{\mathrm{T}}$ are used as parameters of the criterion, one on the source and one on the target language. Full abstraction requires, for source expressions P and Q, that $P \sim_{\mathrm{S}} Q \Leftrightarrow \mathscr{T}(P) \sim_{\mathrm{T}} \mathscr{T}(Q)$. Full abstraction has been criticised as a validity criteria for encodings in [3,23,32]; a historical treatment of the concept can be found in [19, Sect. 18].

An alternative for the equivalence-based validity criteria reviewed above are the ones employing *operational correspondence*, introduced by Nestmann & Pierce in [30]. Here valid encodings are required to satisfy various criteria, differing subtly from paper to paper; often these criteria are chosen to conveniently establish that a given language is or is not as least as expressive as another. Normally some form of operational correspondence is one of these criteria, and as a consequence of this these approaches are suitable for comparing the expressiveness of process calculi with a reduction semantics, rather than system description languages in general. Gorla [22] has selected five of these criteria as a unified approach to encodability and separation results for process calculi—*compositionality*, *name invariance*, *operational correspondence*, *divergence reflection* and *success sensitiveness*—and since then these criteria have been widely accepted as constituting a standard definition of a valid encoding.

In [31] Catuscia Palamidessi employs four requirements for valid encodings between languages that both contain a parallel composition operator |: compositionality, preservation of semantics, a form of name invariance, and the requirement that parallel composition is translated homomorphically, i.e., $\mathscr{T}(P|Q) = \mathscr{T}(P)|\mathscr{T}(Q)$. The latter is not implied by any of the requirements considered above. The justification for this requirement is that it ensures that the translation maintains the degree of distribution of the system. However, Peters, Nestmann & Goltz [35] argue that it is possible to maintain the degree of distribution of a system upon translation without requiring a homomorphic translation of |; in fact they introduce the criterion *preservation of distributability* that is weaker then the homomorphic translation of |.

This paper analyses the encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ of Boudol and Honda & Tokoro of the synchronous into the asynchronous π-calculus, both without the choice operator +. Our aim is to evaluate the validity of these encodings with respect to all criteria for valid encodings summarised above.

Section 2 recalls the encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$. Section 3 reviews the validity criteria from Gorla [22], and recalls the result from [18] that the encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ meet all those criteria. Trivially, $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ also meet Palamidessi's criterion that parallel composition is translated homomorphically, and thus also the criterion on preservation of distributability from [35].

Section 4 focuses on the criterion of compositionality. Gorla's proposal involves a weaker form of this requirement, exactly because encodings like $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ do not satisfy the default form of compositionality. However, we show that these encodings also satisfy a form of compositionality due to [17] that significantly strengthens the one from [22]. Moreover, depending on how the definition of valid encodings between concrete languages generalises to one between parametrised languages, one may even conclude that $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ satisfy the default notion of compositionality.

Section 5 focuses on the criterion of operational correspondence. In [30] two forms of this criterion were proposed, one for *prompt* and one for *nonprompt* encodings. Gorla's form of operational correspondence [22] is the natural common weakening of the forms from Nestmann & Pierce [30], and thus applies to prompt as well as nonprompt encodings. As the encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ are nonprompt, they certainly do not meet the prompt form of operational correspondence from [30]. In [18] it was shown that they not only satisfy the form of [22], but even the nonprompt form from [30].

Gorla's form of operational correspondence, as well as the nonprompt form of [30], weakens the prompt form in two ways. In [18] a natural intermediate form was contemplated that weakens the prompt form in only one of these ways, and the open question was raised whether $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ satisfy this intermediate form of operational correspondence. The present paper answers that question affirmatively.

Gorla's criterion of success sensitiveness is a more abstract form of *barb sensitiveness*. The original barbs were predicates telling whether a process could input or output data over a certain channel. In Section 6 we show that whereas $\mathscr{T}_{\rm B}$ is barb sensitive, $\mathscr{T}_{\rm HT}$ is not. The encoding $\mathscr{T}_{\rm HT}$ becomes barb sensitive if we use a weaker form of barb, abstracting from the difference between input and output. This, however, is against the spirit of the asynchronous π-calculus, where instead one abstracts from input barbs altogether. Gorla's criterion of success sensitiveness thus appears to be an improvement over barb sensitiveness.

Section 7 evaluates $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ under the original validity criterion of Boudol [4], as generalised in [17]; we call a compositional encoding $\mathscr{T}$ *valid up to* a semantic equivalence $\sim$ iff $\mathscr{T}(P) \sim P$ for all source language expressions P. We observe that the encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ are not valid under equivalences that match transition labels, such as early weak bisimilarity, nor under asynchronous weak bisimilarity. Then we show that $\mathscr{T}_{\rm B}$, but not $\mathscr{T}_{\rm HT}$, is valid under weak barbed bisimilarity. This is our main result. Finally, we introduce a new equivalence under which $\mathscr{T}_{\rm HT}$ is valid: a version of weak barbed bisimilarity that drops the distinction between input and output barbs.

Section 8 starts with the result that $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ are both valid under a version of weak barbed bisimilarity where an abstract success predicate takes over the role of barbs. That statement turns out to be equivalent to the statement that these encodings are success sensitive and satisfy a form of operational correspondence that is stronger then Gorla's. One can also incorporate Gorla's requirement of divergence reflection into the definition of form of barbed bisimilarity. Finally, we remark that $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ remain valid when upgrading weak to branching bisimilarity.

Section 9 applies a theorem from [19] to infer from the validity of $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ up to a form of weak barbed bisimilarity, that these encodings are also fully abstract, when taking as source language equivalence weak barbed congruence, and as target language equivalence the congruence closure of that form of weak barbed bisimilarity for the image of the source language within the target language.

2 Encoding Synchrony into Asynchrony

Consider the π-calculus as presented by Milner in [27], i.e., the one of Sangiorgi and Walker [39] without matching, τ-prefixing and choice.

Given a set of *names* $\mathcal{N}$, the set $\mathcal{P}_\pi$ of *processes* or *terms* P of the calculus is given by

$$P ::= \mathbf{0} \quad | \quad \bar{x}z.P \quad | \quad x(y).P \quad | \quad P|Q \quad | \quad (y)P \quad | \quad !P$$

with x, y, z, u, v, w ranging over $\mathcal{N}$.

$\mathbf{0}$ denotes the empty process. $\bar{x}z$ stands for an output guard that sends the name z along the channel x. $x(y)$ denotes an input guard that waits for a name to be transmitted along the channel named x. Upon receipt, the name is substituted for y in the subsequent process. $P|Q$ ($P, Q \in \pi$) denotes a parallel composition between P and Q. $!P$ is the replication construct and $(y)P$ restricts the scope of name y to P.

Definition 1. An occurrence of a name y in π-calculus process $P \in \mathcal{P}_\pi$ is *bound* if it lies within a subexpression $x(y).Q$ or $(y)Q$ of P; otherwise it is *free*. Let $n(P)$ be the set of names occurring in $P \in \mathcal{P}_\pi$, and $fn(P)$ (resp. $bn(P)$) be the set of names occurring free (resp. bound) in P.

Structural congruence, $\equiv$, is the smallest congruence relation on processes satisfying

$$
\begin{array}{lrcl lrcl l}
(1) & P|(Q|R) & \equiv & (P|Q)|R & & (y)\mathbf{0} & \equiv & \mathbf{0} & (5)\\
(2) & P|Q & \equiv & Q|P & & (y)(u)P & \equiv & (u)(y)P & (6)\\
(3) & P|\mathbf{0} & \equiv & P & & (w)(P|Q) & \equiv & P|(w)Q & (7)\\
 & & & & & (y)P & \equiv & (w)P\{^w\!/\!_y\} & (8)\\
(4) & !P & \equiv & P|!P & & x(y).P & \equiv & x(w).P\{^w\!/\!_y\}\ . & (9)
\end{array}
$$

Here $w \notin n(P)$, and $P\{^w\!/\!_y\}$ denotes the process obtained by replacing each free occurrence of y in P by w. Rules (8) and (9) constitute *α-conversion* (renaming of bound names). In case $w \in n(P)$, $P\{^w\!/\!_z\}$ denotes $Q\{^w\!/\!_z\}$ for some process Q obtained from P by means of α-conversion, such that z does not occur within subexpressions $x(w).Q'$ or $(w)Q'$ of Q.

Definition 2. The *reduction relation*, $\longmapsto \subseteq \mathcal{P}_\pi \times \mathcal{P}_\pi$, is generated by the following rules:

$$
\frac{}{\bar{x}z.P|x(y).Q \longmapsto P|Q\{^z\!/\!_y\}} \qquad \frac{P \longmapsto P'}{P|Q \longmapsto P'|Q}
$$

$$
\frac{P \longmapsto P'}{(y)P \longmapsto (y)P'} \qquad \frac{Q \equiv P \quad P \longmapsto P' \quad P' \equiv Q'}{Q \longmapsto Q'}\ .
$$

The asynchronous π-calculus, as introduced by Honda & Tokoro in [24] and by Boudol in [5], is the sublanguage aπ of the fragment π of the π-calculus presented above where all subexpressions $\bar{x}z.P$ have the form $\bar{x}z.\mathbf{0}$, and are written $\bar{x}z$. A characteristic of synchronous communication, as used in π, is that sending a message synchronises with receiving it, so that a process sending a message can only proceed after another party has received it. In the asynchronous π-calculus this feature is dropped, as it is not possible to specify any behaviour scheduled after a send action.

Boudol [5] defines an encoding $\mathscr{T}_{\mathrm{B}}$ from π to aπ inductively as follows:

$$
\begin{array}{rcll}
\mathscr{T}_{\mathrm{B}}(\mathbf{0}) & := & \mathbf{0} & \\
\mathscr{T}_{\mathrm{B}}(\bar{x}z.P) & := & (u)(\bar{x}u|u(v).(\bar{v}z|\mathscr{T}_{\mathrm{B}}(P))) & \text{with } u,v \notin fn(P) \cup \{x,z\}\\
\mathscr{T}_{\mathrm{B}}(x(y).P) & := & x(u).(v)(\bar{u}v|v(y).\mathscr{T}_{\mathrm{B}}(P)) & \text{with } u,v \notin fn(P) \cup \{x\}\\
\mathscr{T}_{\mathrm{B}}(P|Q) & := & (\mathscr{T}_{\mathrm{B}}(P)|\mathscr{T}_{\mathrm{B}}(Q)) & \\
\mathscr{T}_{\mathrm{B}}(!P) & := & !\mathscr{T}_{\mathrm{B}}(P) & \\
\mathscr{T}_{\mathrm{B}}((x)P) & := & (x)\mathscr{T}_{\mathrm{B}}(P) &
\end{array}
$$

always choosing $u \neq v$. To sketch the underlying idea, suppose a π-process is able to perform a communication, for example $\bar{x}z.P|x(y).Q$. In the asynchronous variant of the π-calculus, there is no continuation process after an output operation.

Hence, a translation into the asynchronous π-calculus has to reflect the communication on channel x as well as the guarding role of $\bar{x}z$ for P in the synchronous π-calculus. The idea of Boudol's encoding is to assign a guard to P such that this process must receive an *acknowledgement message* confirming the receipt of z.[1] We write the sender as $P' = (\bar{x}z|u(v).P)$ where $u, v \notin \mathit{fn}(P)$. Symmetrically, the receiver must send the acknowledgement, i. e. $Q' = x(y).(\bar{u}v|Q)$. Unfortunately, this simple transformation is not applicable in every case, because the protocol does not protect the channel u. u should be known to sender and receiver only, otherwise the communication may be interrupted by the environment. Therefore, we restrict the scope of u, and start by sending this private channel to the receiver. The actual message z is now sent in a second stage, over a channel v, which is also made into a private channel between the two processes. The crucial observation is that in $(u)(\bar{x}u|u(v).P^*)$, the subprocess $P^* = \bar{v}z|P$ may only continue after $\bar{x}u$ was accepted by some receiver, and this receiver has acknowledged this by transmitting another channel name v on the private channel u.

The encoding $\mathscr{T}_{\rm HT}$ of Honda & Tokoro [24] differs only in the clauses for the input and output prefix:

$$\begin{array}{rcll}\mathscr{T}_{\rm HT}(\bar{x}z.P) & := & x(u).(\bar{u}z|\mathscr{T}_{\rm HT}(P)) & u \notin \mathit{fn}(P)\cup\{x,z\}\\ \mathscr{T}_{\rm HT}(x(y).P) & := & (u)(\bar{x}u|u(y).\mathscr{T}_{\rm HT}(P)) & u \notin \mathit{fn}(P)\cup\{x\}.\end{array}$$

Unlike Boudol's translation, communication takes place directly after synchronising along the private channel u. The synchronisation occurs in the reverse direction, because sending and receiving messages alternate, meaning that the sending process $\bar{x}z.Q$ is translated into a process that receives a message on channel x and the receiving process $x(y).R$ is translated into a process passing a message on x.

3 Valid Encodings According to Gorla

In [22] a *process calculus* is given as a triple $\mathscr{L} = (\mathcal{P}, \longmapsto, \asymp)$, where

- $\mathcal{P}$ is the set of language terms (called *processes*), built up from k-ary composition operators op,
- $\longmapsto$ is a binary *reduction* relation between processes,
- $\asymp$ is a semantic equivalence on processes.

The operators themselves may be constructed from a set $\mathcal{N}$ of names. In the π-calculus, for instance, there is a unary operator $\bar{x}y.\_$ for each pair of names $x, y \in \mathcal{N}$. This way names occur in processes; the occurrences of names in processes are distinguished in *free* and *bound* ones; $\mathit{fn}(\boldsymbol{P})$ denotes the set of names occurring free in the k-tuple of processes $\boldsymbol{P} = (P_1, \ldots, P_k) \in \mathcal{P}^k$. A *renaming* is a function $\sigma : \mathcal{N} \to \mathcal{N}$; it extends componentwise to k-tuples of names. If $P \in \mathcal{P}$

[1] As observed by a referee, the encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ do not satisfy this constraint: the continuation process P can proceed before z is received. This issue could be alleviated by enriching the protocol with another communication from Q' to P'.

and σ is a renaming, then $P\sigma$ denotes the term P in which each free occurrence of a name x is replaced by $\sigma(x)$, while renaming bound names to avoid name capture.

A k-ary $\mathscr{L}$*-context* $C[\_1, \ldots, \_k]$ is a term build by the composition operators of $\mathscr{L}$ from *holes* $\_1, \ldots, \_k$; the context is called *univariate* if each of these holes occurs exactly once in it. If $C[\_1, \ldots, \_k]$ is a k-ary $\mathscr{L}$*-context* and $P_1, \ldots, P_k \in \mathcal{P}$ then $C[P_1, \ldots, P_k]$ denotes the result of substituting P_i for $\_i$ for each $i{=}1, \ldots, k$.

Let $\Longmapsto$ denote the reflexive-transitive closure of $\longmapsto$. One writes $P \longmapsto^\omega$ if P *diverges*, that is, if there are P_i for $i \in \mathbb{N}$ such that $P = P_0$ and $P_i \longmapsto P_{i+1}$ for all $i \in \mathbb{N}$. Finally, write $P \longmapsto$ if $P \longmapsto Q$ for some term Q.

For the purpose of comparing the expressiveness of languages, a constant $\surd$ is added to each of them [22]. A term P in the upgraded language is said to *report success*, written $P{\downarrow}$, if it has a *top-level unguarded* occurrence of $\surd$.[2] Write $P{\Downarrow}$ if $P \Longmapsto P'$ for a process P' with $P'{\downarrow}$.

Definition 3 ([22]). An *encoding* of $\mathscr{L}_\mathrm{s} = (\mathcal{P}_\mathrm{s}, \longmapsto_\mathrm{s}, \asymp_\mathrm{s})$ into $\mathscr{L}_\mathrm{t} = (\mathcal{P}_\mathrm{t}, \longmapsto_\mathrm{t}, \asymp_\mathrm{t})$ is a pair $(\mathscr{T}, \varphi_\mathscr{T})$ where $\mathscr{T} : \mathcal{P}_\mathrm{s} \to \mathcal{P}_\mathrm{t}$ is called *translation* and $\varphi_\mathscr{T} : \mathcal{N} \to \mathcal{N}^k$ for some $k \in \mathbb{N}$ is called *renaming policy* and is such that for $u \neq v$ the k-tuples $\varphi_\mathscr{T}(u)$ and $\varphi_\mathscr{T}(v)$ have no name in common.

The terms of the source and target languages $\mathscr{L}_\mathrm{s}$ and $\mathscr{L}_\mathrm{t}$ are often called S and T, respectively.

Definition 4 ([22]). An encoding is *valid* if it satisfies the following five criteria.

1. *Compositionality:* for every k-ary operator $\mathtt{op}$ of $\mathscr{L}_\mathrm{s}$ and for every set of names $N \subseteq \mathcal{N}$, there exists a univariate k-ary context $C^N_\mathtt{op}[\_1, \ldots, \_k]$ such that
$$\mathscr{T}(\mathtt{op}(S_1, \ldots, S_k)) = C^N_\mathtt{op}[\mathscr{T}(S_1), \ldots, \mathscr{T}(S_k)]$$
for all $S_1, \ldots, S_k \in \mathcal{P}_\mathrm{s}$ with $\mathit{fn}(S_1, \ldots, S_n) = N$.
2. *Name invariance:* for every $S \in \mathcal{P}_\mathrm{s}$ and $\sigma : \mathcal{N} \to \mathcal{N}$
$$\begin{array}{lll} \mathscr{T}(S\sigma) &= \mathscr{T}(S)\sigma' & \text{if } \sigma \text{ is injective} \\ \mathscr{T}(S\sigma) &\asymp_\mathrm{t} \mathscr{T}(S)\sigma' & \text{otherwise} \end{array}$$
with σ' such that $\varphi_\mathscr{T}(\sigma(a)) = \sigma'(\varphi_\mathscr{T}(a))$ for all $a \in \mathcal{N}$.
3. *Operational correspondence:*
Completeness if $S \Longmapsto_\mathrm{s} S'$ then $\mathscr{T}(S) \Longmapsto_\mathrm{t} \asymp_\mathrm{t} \mathscr{T}(S')$
Soundness if $\mathscr{T}(S) \Longmapsto_\mathrm{t} T$ then $\exists S' : S \Longmapsto_\mathrm{s} S'$ and $T \Longmapsto_\mathrm{t} \asymp_\mathrm{t} \mathscr{T}(S')$.
4. *Divergence reflection:* if $\mathscr{T}(S) \longmapsto^\omega_\mathrm{t}$ then $S \longmapsto^\omega_\mathrm{s}$.

[2] Gorla defines the latter concept only for languages that are equipped with a notion of *structural congruence* $\equiv$ as well as a parallel composition $|$. In that case P has a top-level unguarded occurrence of $\surd$ iff $P \equiv Q|\surd$, for some Q [22]. Specialised to the π-calculus, a *(top-level) unguarded* occurrence is one that not lies strictly within a subterm $\alpha.Q$, where α is τ, $\bar{x}y$ or $x(z)$. For De Simone languages [42], even when not equipped with $\equiv$ and $|$, a suitable notion of an unguarded occurrence is defined in [43].

5. *Success sensitiveness:* $S\Downarrow$ iff $\mathscr{T}(S)\Downarrow$.
   For this purpose $\mathscr{T}(\cdot)$ is extended to deal with the added constant $\surd$ by taking $\mathscr{T}(\surd) = \surd$.

The above treatment of success sensitiveness differs slightly from the one of Gorla [22]. Gorla requires $\surd$ to be a constant of any two languages whose expressiveness is compared. Strictly speaking, this does not allow his framework to be applied to the encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$, as these deal with languages not featuring $\surd$. Here, following [18], we simply allow $\surd$ to be added, which is in line with the way Gorla's framework has been used [21,25,36,34,35,12,13,14,15]. A consequence of this decision is that one has to specify how $\surd$ is translated—see the last sentence of Definition 4—as the addition of $\surd$ to both languages happens after a translation is proposed. This differs from [22], where it is explicitly allowed to take $\mathscr{T}(\surd) \neq \surd$.

In [18] it is established that the encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$, reviewed in Section 2, are valid according to Gorla [22]; that is, both encodings enjoy the five correctness criteria above. Here, the semantic equivalences $\asymp_{\rm s}$ and $\asymp_{\rm t}$ that Gorla assumes to exist on the source and target languages, but were not specified in Section 2, can chosen to be the identity, thus obtaining the strongest possible instantiation of Gorla's criteria. Moreover, the renaming policy required by Gorla as part of an encoding can be chosen to be the identity, taking $k = 1$ in Definition 3. Trivially, $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ also meet Palamidessi's criterion that parallel composition is translated homomorphically, and thus also the criterion on preservation of distributability from [35].

4 Compositionality

Compositionality demands that for every k-ary operator $\mathtt{op}$ of the source language there is a k-ary context $C_{\mathtt{op}}[\_1, \ldots, \_k]$ in the target such that

$$\mathscr{T}(\mathtt{op}(S_1, \ldots, S_k)) = C_{\mathtt{op}}[\mathscr{T}(S_1), \ldots, \mathscr{T}(S_k)]$$

for all $S_1, \ldots, S_k \in \mathcal{P}_{\rm s}$ [4]. Gorla [22] strengthens this requirement by the additional requirement that the context $C_{\mathtt{op}}$ should be univariate; at the same time he weakens the requirement by allowing the required context $C_{\mathtt{op}}$ to depend on the set of names N that occur free in the arguments $S_1, \ldots, S_k$. The application to the encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ shows that we cannot simply strengthen the criterion of compositionality by dropping the dependence on N. For then the present encodings would fail to be compositional. Namely, the context $C_{\bar{x}z.\_}$ depends on the choice of two names u and v, and the choice of these names depends on $N = fn(S_1)$, where S_1 is the only argument of output prefixing. That the choice of $C_{\bar{x}z.\_}$ also depends on x and z is unproblematic.

In [20] a form of compositionality is proposed where $C_{\mathtt{op}}$ does not depend on N, but the main requirement is weakened to

$$\mathscr{T}(\mathtt{op}(S_1, \ldots, S_k)) \stackrel{\alpha}{=} C_{\mathtt{op}}[\mathscr{T}(S_1), \ldots, \mathscr{T}(S_k)].$$

Here $\stackrel{\alpha}{=}$ denotes equivalence up to *α-conversion*, renaming of bound names and variables, for the π-calculus corresponding with rules (8) and (9) of structural congruence. This suffices to rescue the current encodings, for up to α-conversion u and v can always be chosen outside N. It is an open question whether there are examples of intuitively valid encodings that essentially need the dependence of N allowed by [22], i.e., where $C_{\mathsf{op}}^{N_{\mathrm{s}}}$ and $C_{\mathsf{op}}^{N_{\mathrm{t}}}$ differ by more than α-conversion.

Another method of dealing with the fresh names u and v that are used in the encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$, proposed in [17], is to equip the target language with two fresh names that do not occur in the set of names available for the source language. Making the dependence on the choice of set $\mathcal{N}$ of names explicit, this method calls π expressible into aπ if for each $\mathcal{N}$ there exists an $\mathcal{N}'$ such that there is a valid encoding of $\pi(\mathcal{N})$ into a$\pi(\mathcal{N}')$. By this definition, the encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ even satisfy the default definition of compositionality, and its strengthening obtained by insisting the contexts C_{op} to be univariate.

5 Operational Correspondence

Operational completeness (one half of operational correspondence) was formulated by Nestmann & Pierce [30] as

$$S \longmapsto_{\mathrm{s}} S' \text{ then } \mathscr{T}(S) \Longmapsto_{\mathrm{t}} \mathscr{T}(S'). \tag{$\mathfrak{C}$}$$

It makes no difference whether the antecedent of this implication is rephrased as $S \Longmapsto_{\mathrm{s}} S'$, as done by Gorla. Gorla moreover weakens the criterion to

$$S \Longmapsto_{\mathrm{s}} S' \text{ then } \mathscr{T}(S) \Longmapsto_{\mathrm{t}} \asymp_{\mathrm{t}} \mathscr{T}(S'). \tag{$\mathfrak{C}'$}$$

This makes the criterion applicable to many more encodings. In the case of $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$, [18] shows that these encodings not only satisfy ($\mathfrak{C}'$), but even ($\mathfrak{C}$).

Operational soundness also stems from Nestmann & Pierce [30], who proposed two forms of it:

$$\text{if } \mathscr{T}(S) \longmapsto_{\mathrm{t}} T \text{ then } \exists S' \colon S \longmapsto_{\mathrm{s}} S' \text{ and } T \asymp_{\mathrm{t}} \mathscr{T}(S'). \tag{$\mathfrak{I}$}$$
$$\text{if } \mathscr{T}(S) \Longmapsto_{\mathrm{t}} T \text{ then } \exists S' \colon S \Longmapsto_{\mathrm{s}} S' \text{ and } T \Longmapsto_{\mathrm{t}} \mathscr{T}(S'). \tag{$\mathfrak{S}$}$$

The former is meant for "*prompt* encodings, i.e., those where initial steps of literal translations are committing" [30], whereas the latter apply to "nonprompt encodings", that "allow administrative (or book-keeping) steps to precede a committing step". The version of Gorla is the common weakening of ($\mathfrak{I}$) and ($\mathfrak{S}$):

$$\text{if } \mathscr{T}(S) \Longmapsto_{\mathrm{t}} T \text{ then } \exists S' \colon S \Longmapsto_{\mathrm{s}} S' \text{ and } T \Longmapsto_{\mathrm{t}} \asymp_{\mathrm{t}} \mathscr{T}(S'). \tag{$\mathfrak{G}$}$$

It thus applies to prompt as well as nonprompt encodings. The encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ are nonprompt, and accordingly do not meet ($\mathfrak{I}$). In [18] it was shown that they not only satisfy ($\mathfrak{G}$), but even ($\mathfrak{S}$).

An interesting intermediate form between $\mathfrak{I}$ and $\mathfrak{G}$ is

$$\text{if } \mathscr{T}(S) \Longmapsto_{\mathrm{t}} T \text{ then } \exists S' \colon S \Longmapsto_{\mathrm{s}} S' \text{ and } T \asymp_{\mathrm{t}} \mathscr{T}(S'). \tag{$\mathfrak{W}$}$$

Whereas ($\mathfrak{G}$) weakens ($\mathfrak{I}$) in two ways, ($\mathfrak{W}$) weakens ($\mathfrak{I}$) in only one of these ways. Moreover, ($\mathfrak{W}$) is the natural counterpart of ($\mathfrak{C}'$). In [18] the open question was raised whether $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ satisfy ($\mathfrak{W}$), for a reasonable choice of $\asymp_{\rm t}$. (An unreasonable choice, such as the universal relation, tells us nothing.) As pointed out in [18], they do not when taking $\asymp_{\rm t}$ to be the identity relation, or structural congruence.

The present paper answers this question affirmatively, taking $\asymp_{\rm t}$ to be weak barbed bisimilarity. A proof will follow in Section 8.

6 Barb Sensitiveness

Gorla's success predicate is one of the possible ways to provide source and target languages with a set of *barbs* Ω, each being a unary predicate on processes. For $\omega \in \Omega$, write $P{\downarrow_\omega}$ if process P has the barb ω, and $P{\Downarrow_\omega}$ if $P \Longmapsto P'$ for a process P' with $P'{\downarrow_\omega}$. In Gorla's case, $\Omega = \{\surd\}$, and P has the barb $\surd$ iff P has a top-level unguarded occurrence of $\surd$. The standard criterion of barb sensitiveness is then $S{\Downarrow_\omega} \Leftrightarrow \mathscr{T}(S){\Downarrow_\omega}$ for all $\omega \in \Omega$.

A traditional choice of barb in the π-calculus is to take $\Omega = \{x, \bar{x} \mid x \in \mathcal{N}\}$, writing $P{\downarrow_x}$, resp. $P{\downarrow_{\bar{x}}}$, when P has an unguarded occurrence of a subterm $x(z).R$, resp. $\bar{x}y.R$, that lies not in the scope of a restriction operator (x) [26,39]. This makes a barb a predicate that tells weather a process can read or write over a given channel. Boudol's encoding keeps the original channel names of a sending or receiving process invariant. Hence, a translated term does exhibit the same barbs as the source term.

Lemma 1. Let $P \in \mathcal{P}_\pi$ and $a \in \{x, \bar{x} \,|\, x \in \mathcal{N}\}$. Then $P{\downarrow_a}$ iff $\mathscr{T}_{\rm B}(P){\downarrow_a}$.

Proof. With structural induction on P.

- $\mathbf{0}$ and $\mathscr{T}_{\rm B}(\mathbf{0})$ have the same strong barbs, namely none.
- $\bar{x}z.P$ and $\mathscr{T}_{\rm B}(\bar{x}z.P)$ both have only the strong barb $\bar{x}$.
- $x(y).P$ and $\mathscr{T}_{\rm B}(x(y).P)$ both have only strong barb x.
- The strong barbs of $P|Q$ are the union of the ones of P and Q. Using this, the case $P|Q$ follows by induction.
- The strong barbs of $!P$ are the ones of P. Using this, the case $!P$ follows by induction.
- The strong barbs of $(x)P$ are ones of P except x and $\bar{x}$. Using this, the case $(x)P$ follows by induction. □

It follows that $\mathscr{T}_{\rm B}$ meets the validity criterion of barb sensitiveness.

The philosophy behind the asynchronous π-calculus entails that input actions $x(z)$ are not directly observable (while output actions can be observed by means of a matching input of the observer). This leads to semantic identifications like $\mathbf{0} = x(y).\bar{x}y$, for in both cases the environment may observe $\bar{x}z$ only if it supplied $\bar{x}z$ itself first. Yet, these processes differ on their input barbs (${\downarrow_x}$). For this reason, in aπ normally only output barbs ${\downarrow_{\bar{x}}}$ are considered [39]. Boudol's encoding satisfies the criterion of output barb sensitiveness (and in fact also input barb sensitiveness). However, the encoding of Honda & Tokoro does not, as it swaps input and output barbs. As such, it is an excellent example of the benefit of the external barb $\surd$ employed in Gorla's notion of success sensitiveness.

To obtain a weaker form of barb sensitiveness such that also $\mathscr{T}_{\mathrm{HT}}$ becomes barb sensitive, we introduce *channel barbs* $x \in \mathcal{N}$. A process is said to have the channel barb x iff it either has the barb $\bar{x}$ or x. We write $P{\downarrow}^{\mathrm{c}}_{x}$ when P has the channel barb x, and $P \Downarrow^{\mathrm{c}}_{x}$ when a P' exists with $P \Longmapsto P'$ and $P'{\downarrow}^{\mathrm{c}}_{x}$.

Definition 5. An encoding $\mathscr{T}$ is *channel barb sensitive* if $S{\Downarrow}_{\omega} \Leftrightarrow \mathscr{T}(S){\Downarrow}_{\omega}$ for all $\omega \in \Omega$.

This is a weaker criterion than barb sensitiveness, so $\mathscr{T}_{\mathrm{B}}$ is surely channel barb sensitive. It is easy to see that Honda & Tokoro's encoding $\mathscr{T}_{\mathrm{HT}}$, although not barb sensitive, is channel barb sensitive.

7 Validity up to a Semantic Equivalence

This section deals with the original validity criterion from Boudol [4], as generalised in [17]. Following [17] we call a compositional encoding $\mathscr{T}$ *valid up to* a semantic equivalence $\sim\, \subseteq \mathcal{P} \times \mathcal{P}$, where $\mathcal{P} \supseteq \mathcal{P}_{\mathrm{s}} \cup \mathcal{P}_{\mathrm{t}}$, iff $\mathscr{T}(P) \sim P$ for all $P \in \mathcal{P}_{\mathrm{s}}$. A given encoding may be valid up to a coarse equivalence, and invalid up to a finer one. The equivalence for which it is valid is then a measure of the quality of the encoding.

Below, we will evaluate the encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ under a number of semantic equivalences found in the literature. Since these encodings translate a single transition in the source language by a small protocol involving two or three transitions in the target language, they surely will not be valid under *strong* equivalences, demanding step-for-step matching of source transitions by target transitions. Hence we only look at *weak* equivalences.

First we consider equivalences that match transition labels, such as early weak bisimilarity. The encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$ are not valid under such equivalences. Then we show that Boudol's encoding $\mathscr{T}_{\mathrm{B}}$ is valid under weak barbed bisimilarity, and thus certainly under its asynchronous version; however, it is not valid under asynchronous weak bisimilarity. The encoding $\mathscr{T}_{\mathrm{HT}}$ of Honda & Tokoro is not valid under any of these equivalences, but we introduce a new equivalence under which it is valid: a version of weak barbed bisimilarity that drops the distinction between input and output barbs.

7.1 A Labelled Transition Semantics of π

We first present a labelled transition semantics of the (a)synchronous π-calculus, to facilitate the definition of semantic equivalences on these languages. Its labels are drawn from a set of actions $\mathrm{Act} := \{\bar{x}y, x(y), \bar{x}(y) \mid x, y \in \mathcal{N}\} \cup \{\tau\}$. We define free and bound names on transition labels:

$$\text{(OUTPUT-ACT)}\ \frac{}{\bar{x}z.P \xrightarrow{\bar{x}z} P} \qquad \text{(INPUT-ACT)}\ \frac{w \notin fn((y)P)}{x(y).P \xrightarrow{x(w)} P\{^{w}\!/\!_{y}\}}$$

$$\text{(PAR)}\ \frac{P \xrightarrow{\alpha} P' \quad bn(a) \cap fn(Q) = \emptyset}{P|Q \xrightarrow{\alpha} P'|Q} \qquad \text{(COM)}\ \frac{P \xrightarrow{\bar{x}z} P' \quad Q \xrightarrow{x(y)} Q'}{P|Q \xrightarrow{\tau} P'|Q'\{^{z}\!/\!_{y}\}}$$

$$\text{(CLOSE)}\ \frac{P \xrightarrow{\bar{x}(w)} P' \quad Q \xrightarrow{x(w)} Q'}{P|Q \xrightarrow{\tau} (w)(P'|Q')} \qquad \text{(RES)}\ \frac{P \xrightarrow{\alpha} P' \quad y \notin n(a)}{(y)P \xrightarrow{\alpha} (y)P'}$$

$$\text{(OPEN)}\ \frac{P \xrightarrow{\bar{x}y} P' \quad y \neq x \quad w \notin fn((y)P')}{(y)P \xrightarrow{\bar{x}(w)} P'\{^{w}\!/\!_{y}\}} \qquad \text{(REP-ACT)}\ \frac{P \xrightarrow{\alpha} P'}{!P \xrightarrow{\alpha} P'|!P}$$

$$\text{(REP-COMM)}\ \frac{P \xrightarrow{\bar{x}z} P' \quad P \xrightarrow{x(y)} P''}{!P \xrightarrow{\tau} (P'|P''\{^{z}\!/\!_{y}\})|!P} \qquad \text{(REP-CLOSE)}\ \frac{P \xrightarrow{\bar{x}(w)} P' \quad P \xrightarrow{x(w)} P''}{!P \xrightarrow{\tau} ((w)(P'|P''))|!P}$$

Table 1. SOS rules for the synchronous mini-π-calculus. PAR, COM and CLOSE also have symmetric rules.

$$\begin{aligned} fn(\tau) &= \emptyset & bn(\tau) &= \emptyset \\ fn(\bar{x}z) &= \{x, z\} & bn(\bar{x}z) &= \emptyset \\ fn(x(y)) &= \{x\} & bn(x(y)) &= \{y\} \\ fn(\bar{x}(y)) &= \{x\} & bn(\bar{x}(y)) &= \{y\}\ . \end{aligned}$$

For $\alpha \in \text{Act}$ we define $n(\alpha) := bn(\alpha) \cup fn(\alpha)$.

Definition 6. The *labelled transition relation of* π is the smallest relation $\longrightarrow\ \subseteq \mathcal{P}_\pi \times \text{Act} \times \mathcal{P}_\pi$, satisfying the rules of Table 1.

The τ-transitions in the labelled transition semantics play the same role as the reductions in the reduction semantics: they present actual behaviour of the represented system. The transitions with a label different from τ merely represent potential behaviour: a transition $x(y)$ for instance represents the potential of the system to receive a value on channel x, but this potential will only be realised in the presence of a parallel component that sends a value on channel x. Likewise, an output action $\bar{x}z$ or $\bar{x}(y)$ can be realised only in communication with an input action $x(y)$.

The following results show (1) that the labelled transition relations are invariant under structural congruence ($\equiv$), and (2) that the closure under structural congruence of the labelled transition relation restricted to τ-steps coincides with the reduction relation — (2) stems from Milner [27].

Lemma 2 (Harmony Lemma [39, Lemma 1.4.15]).

1. If $P \xrightarrow{\alpha} P'$ and $P \equiv Q$ then $\exists Q'.Q \xrightarrow{\alpha} Q' \equiv P'$
2. $P \longmapsto P'$ iff $\exists P''.P \xrightarrow{\tau} P'' \equiv P'$.

The barbs defined in Section 6 can be characterised in terms of the labelled transition relation as follows:

Remark 1. A process P has a *strong barb on* $x \in \mathcal{N}$, $P{\downarrow_x}$, iff there is a P' with $P \xrightarrow{x(y)} P'$ for some $y \in \mathcal{N}$. It has a *strong barb on* $\bar{x}$, $P{\downarrow_{\bar{x}}}$, iff there is a P' with $P \xrightarrow{\bar{x}z} P'$ or $P \xrightarrow{\bar{x}(z)} P'$ for some $z \in \mathcal{N}$. A process P has a *weak barb on* a ($a \in \{x, \bar{x} \mid x \in \mathcal{N}\}$), $P{\Downarrow_a}$, iff there is a P' such that $P \xrightarrow{\tau}{}^* P'$ and $P'{\downarrow_a}$.

A process P has a *channel barb on* x, $P{\downarrow^{c}_{x}}$, iff it can perform an action on channel x, i. e. iff $P \xrightarrow{\alpha} P'$, for some P', where α has the form $\bar{x}y$, $\bar{x}(y)$ or $x(y)$. Moreover, $P{\Downarrow^{c}_{x}}$ iff a P' exists with $P \xrightarrow{\tau}{}^* P'$ and and $P'{\downarrow^{c}_{x}}$.

7.2 Comparing Transition Labels: Early and Late Weak Bisimilarity

As they make use of intermediate steps (namely the acknowledgement protocol), we must fail proving the validity of the encodings $\mathscr{T}_{\mathrm{B}}$ or $\mathscr{T}_{\mathrm{HT}}$ up to semantics based on transition labels, e. g. *early weak bisimilarity* [39].

Definition 7. A symmetric binary relation $\mathcal{R}$ on π-processes P, Q is a *early weak bisimulation* iff $P \mathrel{\mathcal{R}} Q$ implies

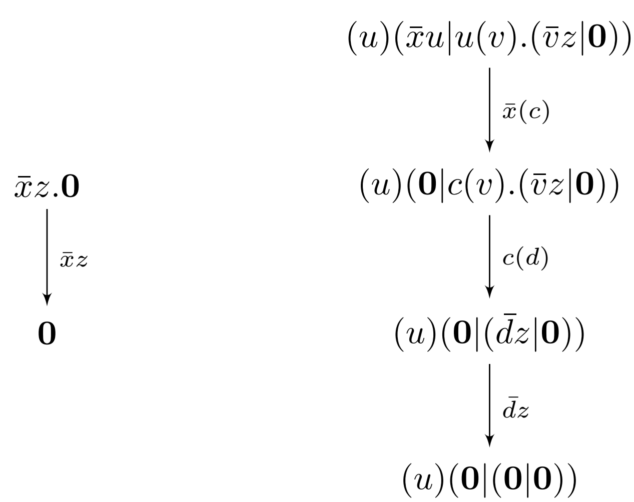
1. if $P \xrightarrow{\tau} P'$ then a Q' exists with $Q \xrightarrow{\tau}{}^* Q'$ and $P' \mathrel{\mathcal{R}} Q'$,
2. if $P \xrightarrow{\alpha} P'$ where $\alpha = \bar{x}z$ or $\bar{x}(y)$ with $y \notin n(P) \cup n(Q)$, then a Q' exists with $Q \xrightarrow{\tau}{}^* \xrightarrow{\alpha} \xrightarrow{\tau}{}^* Q'$ and $P' \mathrel{\mathcal{R}} Q'$,
3. if $P \xrightarrow{x(y)} P'$ with $y \notin n(P) \cup n(Q)$ then for all w a Q' exists satisfying $Q \xrightarrow{\tau}{}^* \xrightarrow{x(y)} \xrightarrow{\tau}{}^* Q'$ and $P'\{^w\!/_y\} \mathrel{\mathcal{R}} Q'\{^w\!/_y\}$.

We denote the largest early weak bisimulation by $\approx_{\mathrm{EWB}}$.

Here $y \notin n(P) \cup n(Q)$ merely ensures the usage of fresh names. A *late* weak bisimulation is obtained by requiring in Clause 3 above that the choice of Q' is independent of w; this gives rise to a slightly finer equivalence relation.

Observation 1. $\mathscr{T}_{\mathrm{B}}$ is not valid up to $\approx_{\mathrm{EWB}}$.

Proof. Let $P = \bar{x}z.\mathbf{0}$ and $\mathscr{T}_{\mathrm{B}}(P) = (u)(\bar{x}u|u(v).(\bar{v}z|\mathbf{0}))$. We present the relevant parts of the labelled transition semantics:



Here, the translated term may perform an input transition $\xrightarrow{c(d)}$ the source term is not capable of. Hence, the processes are not equivalent up to $\approx_{\mathrm{EWB}}$. □

Since late weak bisimilarity is even finer (more discriminating) than $\approx_{\mathrm{EWB}}$, the encoding $\mathscr{T}_{\mathrm{B}}$ is certainly not valid up to late weak bisimilarity. A similar argument shows that neither $\mathscr{T}_{\mathrm{HT}}$ is valid up to early or late weak bisimilarity.

7.3 Weak Barbed Bisimilarity

A weaker approach does not compare all the transitions with visible labels, for these are merely *potential* transitions, that can occur only in certain contexts. Instead it just compares internal transitions, together with the information whether a state has the potential to perform an input or output over a certain channel: the barbs of Section 6. Combining the notion of barbs with the transfer property of classical bisimulation for internal actions only yields *weak barbed bisimilarity* [26]. Here, two related processes simulate each other's internal transitions and furthermore have the same weak barbs.

Definition 8. A symmetric relation $\mathcal{R}$ on $\mathcal{P}_\pi$ is a *weak barbed bisimulation* iff $P\,\mathcal{R}\,Q$ implies

1. if $P{\downarrow_a}$ with $a \in \{x, \bar{x} \mid x \in \mathcal{N}\}$ then $Q{\Downarrow_a}$ and
2. if $P \xrightarrow{\tau} P'$ then a Q' exists with $Q \xrightarrow{\tau}{}^{*} Q'$ and $P'\,\mathcal{R}\,Q'$.

The largest weak barbed bisimulation is denoted by $\stackrel{\bullet}{\approx}$, or $\approx_{\mathrm{WBB}}$.

By Lemma 2 this definition can equivalently be stated with $\longmapsto$ in the role of $\xrightarrow{\tau}$. One of the main results of this paper is that Boudol's encoding is valid up to $\stackrel{\bullet}{\approx}$. The proof of this result is given in the appendix.

7.4 Asynchronous Weak Barbed Bisimilarity

In *asynchronous weak barbed bisimulation* [1], only the names of output channels are observed. Input barbs are ignored here, as it is assumed that an environment is able to observe output messages, but not (missing) inputs.

Definition 9. A symmetric relation S on $\mathcal{P}_\pi$ is an *asynchronous weak barbed bisimulation* iff $P\,\mathcal{R}\,Q$ implies

1. if $P{\downarrow_{\bar{x}}}$, then $Q{\Downarrow_{\bar{x}}}$, and
2. if $P \xrightarrow{\tau} P'$ then a Q' exists with $Q \xrightarrow{\tau}{}^{*} Q'$ and $P'\,\mathcal{R}\,Q'$.

The largest asynchronous weak barbed bisimulation is denoted by $\approx_{\mathrm{AWBB}}$.

Since $\approx_{\mathrm{AWBB}}$ is a coarser equivalence than $\stackrel{\bullet}{\approx}$, we obtain:

Corollary 1. Boudol's encoding is valid up to $\approx_{\mathrm{AWBB}}$.

In [37], a polyadic version of Boudol's encoding was assumed to be valid up to $\approx_{\mathrm{AWBB}}$; see Lemma 17. However, no proof was provided.

7.5 Weak Asynchronous Bisimilarity

We now know that Boudol's translation is valid up to $\approx_{\mathrm{AWBB}}$, but not up to $\approx_{\mathrm{EWB}}$. A natural step is to narrow down this gap by considering equivalences in between. The most prominent semantic equivalence for the asynchronous π-calculus is weak asynchronous bisimilarity, proposed by Amadio et al. [1].

A first strengthening of the requirements for $\approx_{\mathrm{AWBB}}$ is obtained by considering not only output channels but also the messages sent along them.

Definition 10 ([1]). A symmetric relation $\mathcal{R}$ on $\mathcal{P}_\pi$ is a *weak $o\tau$-bisimulation* if $\mathcal{R}$ meets Clauses 1 and 2 (but not necessarily 3) from Definition 7. The largest weak $o\tau$-bisimulation is denoted by $\approx_{\mathrm{W}o\tau}$.

Amadio et al. strengthen this equivalence by adding a further constraint for input transitions.

Definition 11 ([1]). A relation $\mathcal{R}$ is a *weak asynchronous bisimulation* iff $\mathcal{R}$ is a weak $o\tau$-bisimulation such that $P \mathrel{\mathcal{R}} Q$ and $P \xrightarrow{\tau}{}^{*} \xrightarrow{x(y)} \xrightarrow{\tau}{}^{*} P'$ implies

- either a Q' exists satisfying a condition akin to Clause 3 of Definition 7,
- or a Q' exists such that $Q \xrightarrow{\tau}{}^{*} Q'$ and $P' \mathrel{\mathcal{R}} (Q' | \bar{x}y)$.

The largest weak asynchronous bisimulation is denoted by $\approx_{\mathrm{WAB}}$.

Observation 2. Boudol's translation $\mathscr{T}_{\mathrm{B}} : \mathcal{P}_\pi \to \mathcal{P}_{\mathrm{a}\pi}$ is not valid up to $\approx_{\mathrm{W}o\tau}$, and thus not up to $\approx_{\mathrm{WAB}}$.

Proof. Consider the proof of Observation 1. $\bar{x}z.\mathbf{0}$ sends a free name along x while $(u)(\bar{x}u|u(v).(\bar{v}z|\mathbf{0}))$ sends a bound name along the same channel. Since $\approx_{\mathrm{W}o\tau}$ differentiates between free and bound names, the transition systems of $\bar{x}z$ and its translation are not $\approx_{\mathrm{W}o\tau}$-equivalent. □

7.6 Weak Channel Bisimilarity

From the equivalences considered, weak barbed bisimilarity, $\overset{\bullet}{\approx}$, is the finest one that supports the validity of Boudol's translation. However, it does not validate Honda and Tokoro's translation.

Observation 3. Honda and Tokoro's translation $\mathscr{T}_{\mathrm{HT}}$ is not valid up to $\approx_{\mathrm{AWBB}}$, and thus not up to $\overset{\bullet}{\approx}$, $\approx_{\mathrm{W}o\tau}$, or $\approx_{\mathrm{WAB}}$.

Proof. Let $P = \bar{x}z.\mathbf{0}$. Then $P{\downarrow_{\bar{x}}}$. The translation is $\mathscr{T}_{\mathrm{HT}}(P) = x(u).(\bar{u}z|\mathbf{0})$ and $\mathscr{T}_{\mathrm{HT}}(P){\not\Downarrow_{\bar{x}}}$. □

To address this problem we introduce an equivalence even weaker than $\overset{\bullet}{\approx}$, which does not distinguish between input and output channels.

Definition 12. A symmetric relation $\mathcal{R}$ on $\mathcal{P}_\pi$ is a *weak channel bisimulation* if $P \mathrel{\mathcal{R}} Q$ implies

1. if $P \downarrow_x^{\mathrm{c}}$ then $Q \Downarrow_x^{\mathrm{c}}$ and
2. if $P \stackrel{\tau}{\longrightarrow} P'$ then a Q' exists with $Q \stackrel{\tau}{\longrightarrow}^* Q'$ and $P' \mathcal{R} Q'$.

The largest weak channel bisimulation is denoted $\approx_{\mathrm{WCB}}$.

Theorem 1. Honda and Tokoro's encoding $\mathscr{T}_{\mathrm{HT}}$ is valid up to $\approx_{\mathrm{WCB}}$.

The proof is similar to the one of Theorem 4. Here we use that Lemmas 6 and 7 also apply to $\mathscr{T}_{\mathrm{HT}}$ [18] and Lemma 1 now holds with $\downarrow_x^{\mathrm{c}}$ in the role of $\downarrow_a$.

Since $\approx_{\mathrm{WCB}}$ is a coarser equivalence than $\stackrel{\bullet}{\approx}$, we also obtain that Boudol's translation is valid up to $\approx_{\mathrm{WCB}}$.

7.7 Overview

We thus obtain the following hierarchy of equivalence relations on π-calculus processes (cf. Fig. 1), with the vertical lines indicating the realm of validity of $\mathscr{T}_{\mathrm{HT}}$ and $\mathscr{T}_{\mathrm{B}}$, respectively.

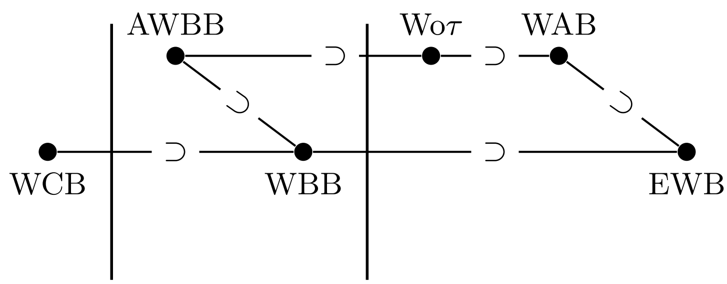


Fig. 1. A hierarchy on semantic equivalence relations for π-calculus processes, with separation lines indicating where the encodings discussed in this paper pass and fail validity.

8 Validity up to an Equivalence versus Validity à la Gorla

The idea of introducing a success predicate $\surd$ to the source and target language of an encoding, as implicit in Gorla's criterion of success sensitiveness, can be applied to the equivalence based approach as well.

Definition 13. Let $\mathcal{P}_{\mathrm{s}}$, $\mathcal{P}_{\mathrm{t}}$ be languages equipped with a reduction relation $\longmapsto$, and $\mathcal{P}_{\mathrm{s}}^{\surd}$, $\mathcal{P}_{\mathrm{t}}^{\surd}$ their extensions with a success predicate $\surd$. A symmetric relation $\mathcal{R}$ on $\mathcal{P} := \mathcal{P}_{\mathrm{s}}^{\surd} \uplus \mathcal{P}_{\mathrm{t}}^{\surd}$ is a *success respecting weak reduction bisimulation* if $P \mathcal{R} Q$ implies

1. if $P{\downarrow_{\surd}}$ then $Q{\Downarrow_{\surd}}$ and
2. if $P \longmapsto P'$ then a Q' exists with $Q \Longmapsto Q'$ and $P' \mathcal{R} Q'$.

The largest success respecting weak reduction bisimulation is denoted $\stackrel{\bullet}{\approx}^{\surd}$.

An compositional encoding $\mathscr{T} : \mathcal{P}_{\mathrm{s}} \rightarrow \mathcal{P}_{\mathrm{t}}$ is *valid up to* $\stackrel{\bullet}{\approx}^{\surd}$ if its extension $\mathscr{T}^{\surd} : \mathcal{P}_{\mathrm{s}}^{\surd} \rightarrow \mathcal{P}_{\mathrm{t}}^{\surd}$, defined by $\mathscr{T}^{\surd}(\surd) := \surd$, satisfies $\mathscr{T}^{\surd}(P) \stackrel{\bullet}{\approx}^{\surd} P$ for all $P \in \mathcal{P}_{\mathrm{s}}^{\surd}$.

Trivially, a variant of Lemma 1 with $\surd$ in the role of a holds for $\mathscr{T}_{\rm B}$ as well as $\mathscr{T}_{\rm HT}$: we have $P{\downarrow_\surd}$ iff $\mathscr{T}_{\rm B}(P){\downarrow_\surd}$ iff $\mathscr{T}_{\rm HT}(P){\downarrow_\surd}$. Using this, the material in the appendix implies that:

Theorem 2. The encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ are valid up to $\stackrel{\bullet}{\approx}^\surd$. □

This approach has the distinct advantage over dealing with input and output barbs that both encodings are seen to be valid without worrying on what kinds of barbs to use exactly.

The following correspondence between operational correspondence, success sensitivity and validity up to $\stackrel{\bullet}{\approx}^\surd$ was observed in [33], and not hard to infer from the definitions.

Theorem 3. An encoding $\mathscr{T}$ is success sensitive and satisfies operational correspondence criteria ($\mathfrak{C}'$) and ($\mathfrak{W}$), taking $\asymp$ to be $\stackrel{\bullet}{\approx}^\surd$, iff it is valid up to $\stackrel{\bullet}{\approx}^\surd$. □

This yields the result promised in Section 5:

Corollary 2. The encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ satisfy criterion ($\mathfrak{W}$).

The validity of $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ by Gorla's criteria, established in [18], by the analysis of [33], already implied that $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ are valid up to *success respecting coupled reduction similarity* [33], a semantic equivalence strictly coarser than $\stackrel{\bullet}{\approx}^\surd$. Theorem 2 yields a nontrivial strengthening of that result.

Gorla's criterion of divergence reflection can be strengthened to *divergence preservation* by requiring

$$\mathscr{T}(S) \longmapsto_{\rm t}^\omega \;\Leftrightarrow\; S \longmapsto_{\rm s}^\omega \;;$$

by [18, Remark 1] this criterion is satisfied by $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ as well. A bisimulation $\mathcal{R}$ is said to preserve divergence iff $P \mathrel{\mathcal{R}} Q$ implies $P \longmapsto_{\rm t}^\omega \;\Leftrightarrow\; Q \longmapsto_{\rm s}^\omega$; the largest divergence preserving, success respecting weak reduction bisimulation is denoted $\stackrel{\bullet}{\approx}^{\surd\Delta}$. As observed in [33], Theorem 3 can be extended as follows with divergence preservation:

Observation 4. An encoding $\mathscr{T}$ is success sensitive, divergence preserving, and satisfies operational correspondence criteria ($\mathfrak{C}'$) and ($\mathfrak{W}$), taking $\asymp$ to be $\stackrel{\bullet}{\approx}^{\surd\Delta}$, iff it is valid up to $\stackrel{\bullet}{\approx}^{\surd\Delta}$. □

Hence, $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ are valid up to $\stackrel{\bullet}{\approx}^{\surd\Delta}$. This statement implies all criteria of Gorla, except for name invariance.

In [19, Definition 26] the notion of *divergence preserving branching barbed bisimilarity* is defined. This definition is parametrised by the choice of barbs; when taking the success predicate $\surd$ as only barb, it could be called *divergence preserving, success respecting branching reduction bisimilarity*. It is strictly finer then $\stackrel{\bullet}{\approx}^{\surd\Delta}$. It is not hard to adapt the proof of Theorem 4 in the appendix to show that $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ are even valid up to this equivalence.

9 Full Abstraction

The criterion of full abstraction is parametrised by the choice of two semantic equivalences $\sim_{\rm S}$ and $\sim_{\rm T}$, one on the source and one on the target language. It requires, for source expressions P and Q, that $P \sim_{\rm S} Q \Leftrightarrow \mathscr{T}(P) \sim_{\rm T} \mathscr{T}(Q)$.

It is well known that the encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ fail to be fully abstract w.r.t. $\cong^c$ and $\cong^c_a$. Here $\cong^c$ is *weak barbed congruence*, the congruence closure of $\dot{\approx}^{\surd}$ (or $\approx_{\rm AWBB}$) on the source language, and $\cong^c_a$ is *asynchronous weak barbed congruence*, the congruence closure of $\dot{\approx}^{\surd}$ (or $\approx_{\rm AWBB}$) on the target language. These are often deemed to be the most natural semantic equivalences on π and aπ. The well-known counterexample is given by the π processes $\bar{x}z|\bar{x}z$ and $\bar{x}z.\bar{x}z$. Although related by $\cong^c$, their translations are not related by $\cong^c_a$.

In [10] this problem is addressed by proposing a strict subcalculus $SA\pi$ of the target language that contains the image of the source language under of a version Honda & Tokoro's encoding, such that this encoding is fully abstract w.r.t. $\cong^c$ and the congruence closure of $\dot{\approx}^{\surd}$ (or $\approx_{\rm AWBB}$) w.r.t. $SA\pi$. In [37] a similar solution to the same problem was found earlier, but for a variant of Boudol's encoding from the *polyadic* π-calculus to the (monadic) asynchronous π-calculus. They define a class of *well-typed* expressions in the asynchronous π-calculus, such that the well-typed expressions constitute a subcalculus of the target language that contains the image of the source language under the encoding. Again, the encoding is fully abstract w.r.t. $\cong^c$ and the congruence closure of $\dot{\approx}^{\surd}$(or $\approx_{\rm AWBB}$) w.r.t. that sublanguage.

By [20, Theorem 4] such results can always be achieved, namely by taking as target language exactly the image of the source language under the encoding. In this sense a full abstraction result is a direct consequence of the validity of the encodings up to $\dot{\approx}^{\surd}$, taking for $\sim_{\rm S}$ the congruence closure of $\dot{\approx}^{\surd}$ w.r.t. the source language, and for $\sim_{\rm T}$ the congruence closure of $\dot{\approx}^{\surd}$ w.r.t. the image of the source language within the target language. What the results of [37,10] add is that the sublanguage may be strictly larger than the image of the source language, and that its definition is not phrased in terms of the encoding.

10 Conclusion

We examined which of the quality criteria for encodings from the literature support the validity of the well-known encodings $\mathscr{T}_{\rm B}$ and $\mathscr{T}_{\rm HT}$ of the asynchronous into the synchronous π-calculus. It was already known [18] that these encodings are valid à la Gorla [22]; this implies that they are valid up to success respecting coupled reduction similarity [33]. We strengthened this result by showing that they are even valid up to divergence preserving, success respecting weak reduction bisimilarity. That statement implies all criteria of Gorla, except for name invariance. Moreover, it implies a stronger form of operation soundness then considered by Gorla, namely

$$\text{if } \mathscr{T}(S) \Longmapsto_{\rm t} T \text{ then } \exists S'\colon S \Longmapsto_{\rm s} S' \text{ and } T \asymp_{\rm t} \mathscr{T}(S'). \qquad (\mathfrak{W})$$

Crucial for all these results is that we employ Gorla's external barb $\surd$, a success predicate on processes. When reverting to the internal barns x and $\bar{x}$ commonly used in the π-calculus, we see a potential difference in quality between the encodings $\mathscr{T}_{\mathrm{B}}$ and $\mathscr{T}_{\mathrm{HT}}$. Boudol's translation $\mathscr{T}_{\mathrm{B}}$ is valid up to weak barbed bisimilarity, regardless whether all barbs are used, or only output barbs $\bar{x}$. However, Honda and Tokoro's translation $\mathscr{T}_{\mathrm{HT}}$ is not valid under either of these forms of weak barbed bisimilarity. In order to prove the validity of $\mathscr{T}_{\mathrm{HT}}$, we had to use the novel weak channel bisimilarity that does not distinguish between input and output channels. Conversely, we conjecture that there is no natural equivalence for which $\mathscr{T}_{\mathrm{HT}}$ is valid, but $\mathscr{T}_{\mathrm{B}}$ is not. Hence, Honda and Tokoro's encoding can be regarded as weaker than the one of Boudol. Whether $\mathscr{T}_{\mathrm{B}}$ is to be preferred, because it meets stronger requirements/equivalences, is a decision that should be driven by the requirements of an application the encoding is used for.

The validity of $\mathscr{T}_{\mathrm{B}}$ under semantic equivalences has earlier been investigated in [7,8], In [7] it is established that $\mathscr{T}_{\mathrm{B}}$ is valid up to *may testing* [9] and *fair testing equivalence* [6,28]. Both results now follow from Theorem 2, since may and fair testing equivalence are coarser then $\stackrel{\bullet}{\approx}^{\surd}$. On the other hand, [7] also shows that $\mathscr{T}_{\mathrm{B}}$ is not valid up to a form of *must testing*; in [8] this result is strengthened to pertain to any encoding of π into aπ. It follows that this form of must testing equivalence is not implied by $\stackrel{\bullet}{\approx}^{\surd}$, and not even by $\stackrel{\bullet}{\approx}^{\surd\Delta}$.

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Appendix: Boudol's Translation is Valid up to $\stackrel{\bullet}{\approx}$

Before we prove validity of Boudol's translation up to weak barbed bisimulation, we further investigate the protocol steps established by Boudol's encoding. Let $P' = \bar{x}z.P$ and $Q' = x(y).Q$. Pick u, v not free in P and Q, with $u \neq v$. Write $P^* := \bar{v}z|\mathscr{T}_{\mathrm{B}}(P)$ and $Q^* := v(y).\mathscr{T}_{\mathrm{B}}(Q)$. Then

$$\begin{aligned}\mathscr{T}_{\mathrm{B}}(P'|Q') &= (u)(\bar{x}u|u(v).P^*) \mid x(u).(v)(\bar{u}v|Q^*)\\ &\longmapsto (u)\big(u(v).P^* \mid (v)(\bar{u}v|Q^*)\big)\\ &\longmapsto (v)(P^*|Q^*)\\ &\longmapsto \mathscr{T}_{\mathrm{B}}(P)|(\mathscr{T}_{\mathrm{B}}(Q)\{z/y\})\ .\end{aligned}$$

Here structural congruence is applied in omitting parallel components **0** and empty binders (u) and (v). Now the crucial idea in our proof is that the last two reductions are *inert*, in that set of the potential behaviours of a process is not diminished by doing (internal) steps of this kind. The first reduction above in general is not inert, as it creates a commitment between a sender and a receiver to communicate, and this commitment goes at the expense of the potential of one of the two parties to do this communication with another partner. We employ a relation that captures these inert reductions in a context.

Definition 14 ([18]). Let $\Longrightarrow$ be the smallest relation on $\mathcal{P}_{\mathrm{a}\pi}$ such that

1. $(v)(\bar{v}y|P|v(z).Q) \Longrightarrow P|(Q\{y/z\})$,
2. if $P \Longrightarrow Q$ then $P|C \Longrightarrow Q|C$,
3. if $P \Longrightarrow Q$ then $(w)P \Longrightarrow (w)Q$,
4. if $P \equiv P' \Longrightarrow Q' \equiv Q$ then $P \Longrightarrow Q$,

where $v \notin fn(P) \cup fn(Q\{y/z\})$.

First of all observe that whenever two processes are related by $\Longrightarrow$, an actual reduction takes place.

Lemma 3 ([18]). If $P \Longrightarrow Q$ then $P \longmapsto Q$.

The next two lemmas confirm that inert reductions do not diminish the potential behaviour of a process.

Lemma 4 ([18]). If $P \Longrightarrow Q$ and $P \longmapsto P'$ with $P' \not\equiv Q$ then there is a Q' with $Q \longmapsto Q'$ and $P' \Longrightarrow Q'$.

Corollary 3. If $P \Longrightarrow^* Q$ and $P \longmapsto P'$ then either $P' \Longrightarrow^* Q$ or there is a Q' with $Q \longmapsto Q'$ and $P' \Longrightarrow^* Q'$.

Proof. By repeated application of Lemma 4. □

Lemma 5. If $P \Longrightarrow Q$ and $P{\downarrow_a}$ for $a \in \{x, \bar{x} \mid x \in \mathcal{N}\}$ then $Q{\downarrow_a}$.

Proof. Let $(\tilde{w})P$ for $\tilde{w} = \{w_1, \ldots, w_n\} \subseteq \mathcal{N}$ with $n \in \mathbb{N}$ denote $(w_1) \cdots (w_n)P$ for some arbitrary order of the (w_i). Using a trivial variant of Lemma 1.2.20 in [39], there are $\tilde{w} \subseteq \mathcal{N}$, $x, y, z \in \mathcal{N}$ and $R, C \in \mathcal{P}_{\mathrm{a}\pi}$, such that $x \in \tilde{w}$ and $P \equiv (\tilde{w})((\bar{x}y|x(z).R)|C) \longmapsto (\tilde{w})((\mathbf{0}|R\{y/z\})|C) \equiv Q$. Since $P{\downarrow_a}$, it must be that $a{=}u$ or $\bar{u}$ with $u \notin \tilde{w}$, and $C{\downarrow_a}$. Hence $Q{\downarrow_a}$. □

The following lemma states, in terms of Gorla's framework, *operational completeness* [22]: if a source term is able to make a step, then its translation is able to simulate that step by protocol steps.

Lemma 6 ([18]). *Let $P, P' \in \mathcal{P}_\pi$. If $P \longmapsto P'$ then $\mathscr{T}_{\mathrm{B}}(P) \longmapsto^* \mathscr{T}_{\mathrm{B}}(P')$.*

Finally, the next lemma was a crucial step in establishing *operational soundness* [22].

Lemma 7 ([18]). Let $P \in \mathcal{P}_\pi$ and $Q \in \mathcal{P}_{\mathrm{a}\pi}$. If $\mathscr{T}_{\mathrm{B}}(P) \longmapsto Q$ then there is a P' with $P \longmapsto P'$ and $Q \Longrightarrow^* \mathscr{T}_{\mathrm{B}}(P')$.

Using these lemmas, we prove the validity of Boudol's encoding up to weak barbed bisimilarity.

Theorem 4. Boudol's encoding is valid up to $\stackrel{\bullet}{\approx}$.

Proof. Define the relation $\mathcal{R}$ by $P \mathrel{\mathcal{R}} Q$ iff $Q \Longrightarrow^* \mathscr{T}_{\mathrm{B}}(P)$. It suffices to show that the symmetric closure of $\mathcal{R}$ is a weak barbed bisimulation.

To show that $\mathcal{R}$ satisfies Clause 1 of Definition 8, suppose $P \mathrel{\mathcal{R}} Q$ and $P{\downarrow_a}$ for $a \in \{x, \bar{x} \,|\, x \in \mathcal{N}\}$. Then $\mathscr{T}_{\mathrm{B}}(P){\downarrow_a}$ by Lemma 1. Since $Q \Longrightarrow^* \mathscr{T}_{\mathrm{B}}(P)$, we obtain $Q \longmapsto^* \mathscr{T}_{\mathrm{B}}(P)$ by Lemma 3, and thus $Q{\Downarrow_a}$.

To show that $\mathcal{R}$ also satisfies Clause 2, suppose $P \mathrel{\mathcal{R}} Q$ and $P \longmapsto P'$. Since $Q \Longrightarrow^* \mathscr{T}_{\mathrm{B}}(P)$, by Lemmas 3 and 6 we have $Q \longmapsto^* \mathscr{T}_{\mathrm{B}}(P) \longmapsto^* \mathscr{T}_{\mathrm{B}}(P')$, and also $P' \mathrel{\mathcal{R}} \mathscr{T}_{\mathrm{B}}(P')$.

To show that $\mathcal{R}^{-1}$ satisfies Clause 1, suppose $P \mathrel{\mathcal{R}} Q$ and $Q{\downarrow_a}$. Since $Q \Longrightarrow^* \mathscr{T}_{\mathrm{B}}(P)$, Lemma 5 yields $\mathscr{T}_{\mathrm{B}}(P){\downarrow_a}$, and Lemma 1 gives $P{\downarrow_a}$, which implies $P{\Downarrow_a}$.

To show that $\mathcal{R}^{-1}$ satisfies Clause 2, suppose $P \mathrel{\mathcal{R}} Q$ and $Q \longmapsto Q'$. Since $Q \Longrightarrow^* \mathscr{T}_{\mathrm{B}}(P)$, by Corollary 3 either $Q' \Longrightarrow^* \mathscr{T}_{\mathrm{B}}(P)$ or there is a Q'' with $\mathscr{T}_{\mathrm{B}}(P) \longmapsto Q''$ and $Q' \Longrightarrow^* Q''$. In the first case $P \mathrel{\mathcal{R}} Q'$, so taking $P' := P$ we are done. In the second case, by Lemma 7 there is a P' with $P \longmapsto P'$ and $Q'' \Longrightarrow^* \mathscr{T}_{\mathrm{B}}(P')$. We thus have $P' \mathrel{\mathcal{R}} Q'$. □